AN APPLICATION OF QUEUEING THEORY TO CARRIER AIRCRAFT LANDING DELAYS

PETER RIPPA

Library
U. S. Naval Postgraduate School
Monterey, California

DIRECTIONS FOR BINDING

BIND IN
(CIRCLE ONE)

BUCKRAM
COLOR NO. 8854

FABRIKOID
COLOR__________

LEATHER
COLOR__________

OTHER INSTRUCTIONS

LETTERING ON BACK TO BE EXACTLY AS PRINTED HERE.

RIPPA

1954

Thesis
R575s

letter on the front cover:

AN APPLICATION OF QUEUEING THEORY
TO CARRIER AIRCRAFT LANDING DELAYS

PETER RIPPA

AN APPLICATION OF QUEUEING THEORY
TO CARRIER AIRCRAFT LANDING DELAYS

by

Peter Rippa

Lieutenant, United States Navy

Submitted in partial fulfillment
of the requirements
for the degree of
MASTER OF SCIENCE

UNITED STATES NAVAL POSTGRADUATE SCHOOL
Monterey, California
1 9 5 4

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

from the

United States Naval Postgraduate School

ABSTRACT

A brief summary of the literature pertaining to the theory of queues is presented to acquaint the reader with the mathematical techniques available.

A quantitative analysis of the landing process around an aircraft carrier is made with a view towards a more efficient utilization of the landing platform. The theory of queues supplies the mathematical techniques, thus making it possible to come to grips analytically with the problem of reducing the landing time.

The problem has been idealized severely but is applicable under certain conditions to the actual landing process. The results may be used for investigating the distribution of the means of (a) the aircraft waiting time, (b) the time wasted by the aircraft carrier during the landing operations, and (c) the number of aircraft waiting provided we know (or assume we know):

(i) the minimum safe landing interval,

(ii) the distribution of the landing times (landing rate),

(iii) the distribution of the time intervals between aircraft entering the landing pattern (departure rate).

If we have not assumed the above, this information must be based on data obtained from observation of actual landing operations.

A method is also indicated whereby the problem may be analyzed by a quasi-emperical technique. This is a well known substitute for an analytical treatment if the mathematics become intractable.

PREFACE

With the advent of the jet age, terminal congestion and resulting aircraft delays are of increasing importance in all branches of aviation. The scientific approach provides a means for predicting how these delays will be affected by varying conditions, and so makes possible an economic valuation of efforts directed towards reducing these delays.

Operational research methods have a threefold application in this connection: (a) in ascertaining the wide field of practical problems susceptible to treatment; (b) in obtaining the basic data needed to enable the delay problems to be specified; (c) in selecting appropriate solutions.

Queueing theory supplies certain mathematical techniques whereby the operations analyst can come to grips analytically with the problem of aircraft delays. Even where mathematics become intractable, there are good prospects of empirical solutions by means of electronic calculators.

The purpose of this paper is to apply queueing theory to a problem of some importance in naval aviation, carrier aircraft landing delays. It follows that the results may allow a more efficient utilization of the landing platform. Two things are of particular interest in the general queueing problem, the size of the queue (stack) and the waiting time of the customer (aircraft delay).

Chapter 1 gives an analysis of the general queueing problem and

certain technical implications which, while they are not strictly integral to the problem, throw considerable light on the report. Chapter 11 presents a detailed analysis of one aspect of the carrier aircraft problem, a summary of the results, the conclusions, and further considerations of statistical implications.

This study was undertaken at the United States Naval Postgraduate School during the latter half of the academic year 1953-54. The writer is deeply indebted to Professor Charles C. Torrance whose constant criticism and encouragement were invaluable. To him go my foremost thanks. The writer is also indebted to Professor A. Boyd Mewborn for reading the paper and for making valuable suggestions and remarks.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

CHAPTER I

QUEUEING THEORY

1.1 Introduction to the Queueing Problem.

The study of queues has been of interest to mathematicians and engineers for the past forty years. Queueing occurs when demands for service are greater than availability of service, and it becomes necessary to postpone the demands by a system of queueing or marshalling. While queueing is a natural process, marshalling is an attempt to do something about it. Queueing, however, is essentially a temporary phenomenon, as otherwise the queue would grow indefinitely in size. Thus one of the first problems in any queueing situation is to determine whether or not the demand will outstrip the service mechanism. When this does not happen, so that the queue is continually returning to zero size, then the situation is said to be in equilibrium, or, in statistical terminology, the process is stationary. When the demand equals the service capabilities the situation becomes very delicately balanced; a slight reduction in demand yields a sensible queueing system in equilibrium, while a slight increase results in an ever lengthening queue.

The theory of queues has a special appeal for the scientist interested in stochastic processes, for under certain conditions it provides an example which is both stationary and non-Markovian. Thus one may expect in studying the queue to gain insight into other stochastic phenomena, and to acquire a valuable facility in the handling of the relevant techniques. In addition, the theory has an astonishing range

of applications. A current limitation of the theory is that the number of papers that deal with the theoretical side of the subject is small. And unfortunately those that exist are rather heavy reading.

1.2 Queueing system.

The simple queueing system is concerned not so much with the matching of supply and demand, but with a series of customers demanding service at a single counter and waiting in turn to be served. Customer is here used in a technical sense; in the present paper it should be equated with aircraft. To complete the specification (Kendall 2) one must give a careful account of:

(a) the input-process,

(b) the queue-discipline,

(c) the service-mechanism.

1.2.1 Input-process.

The input is random whenever the chance or probability that a customer will arrive at any instant of time is the same for all instants past, present, and future, and also is unaffected by any past or future arrivals. The random input is often called a "poisson input" because of the use of the Poisson distribution to describe the arrivals. A general independent input (Lindley 3) is one where the intervals between the arrivals of successive customers are independent with identical probability distributions, but this distribution is arbitrary. The input is regular when the customers arrive at a constant rate. The input is correlated whenever the intervals between arrivals are not independent.

1.2.2 Queue-discipline.

The queue-discipline is the rule or moral code determining the

manner in which the customers form up into a queue, and the manner in which they behave while waiting. In the simplest case they line up before the single counter and await their respective turns. The theory of multiple queues or many servers seems, except under simplifying assumptions which do not always correspond to reality, to be a problem of considerable difficulty and will not be considered here.

1.2.3 Service-mechanism.

The service-mechanism may be described as the mode of service. In this connection the service time is of basic importance and is the time that elapses while a particular customer is being served.

1.3 Traffic density.

A parameter of special importance is the traffic intensity or density ϵ, defined as the ratio of the average arrival rate to the average service rate. This may also be expressed as the ratio of the mean service time to the mean arrival interval.

1.4 Markovian properties.

Before the mathematical theory is presented, an interesting qualitative feature of the problem should be commented on.

Suppose that the state of a stochastic system at time t is described by a random function X(t); the stochastic process is then said to be of Markov type (Doob 9) if a knowledge of the present value of X(t) makes all information about its past history irrelevant to a prediction of its future behaviour. Such a process may become non-Markovian if part of the information contained in X(t) is suppressed, for a knowledge of the previous behaviour may enable some of the suppressed information about

the present state to be recovered. The queueing process with a Poissonian input and a single server is Markovian if the present state of the process is described by the pair of random variables q and v_o, where q is the instantaneous queue size and v_o is the expended service time of the customer at the head of the queue; in general it ceases to be Markovian if the state of the process is measured by the queue size alone (Kendall 2). The only exception to this statement occurs when the service time has a negative exponential distribution.

1.5 Poisson input.

Kendall (2) gives a general review of some points in congestion theory. A presentation is made of the Pollaczek "equilibrium" theory for the single counter queue fed by an input of the Poisson type and associated with a general service time distribution. It is pointed out that although the stochastic processes describing the fluctuations in queue size is not in general Markovian, it is possible to work instead with an enumerable Markov chain if attention is directed to the epochs at which individual customers depart. The ergodic properties of the chain are investigated with the aid of Feller's (8) theory of recurrent events.

The simplest hypothesis about the input is one which states that the customers arrive "at random", the number of arrivals in time T being a Poisson variable of expectation T/a, where a is the mean arrival interval. The time interval t between two consecutive arrivals will then have the negative exponential distribution

$$dF(t) = e^{-t/a}\frac{dt}{a}, \quad 0 < t < \infty. \qquad 1.1$$

This is seen if one recalls that dF(t) is simply the probability of

no arrivals in a time interval t. The successive t-variables will be statistically independent. This is the hypothesis adopted by Kendall.

1.5.1 Mean waiting time with general service time distribution.

Let the queue be of the type with a single server fed by a Poissonian input and let the traffic intensity ϵ be less than unity, so that the system is not saturated. Let q be the size of the queue which the departing customer leaves behind him, not including himself, but including the next person to be served (q may be zero). The next person to be served has a service time v $\left[E(v) = b\right]$, say, and suppose that during this time r new customers arrive. Then conditionally r is a Poisson variable of mean value v/a, for given v, and v has the service time distribution. Let q' be the size of the queue which the next person leaves behind him.

Assume that statistical equilibrium exists. Then the random variables q and q' have the same marginal distribution, and in particular their mean and mean square values must be equal and finite:

$$E(q) = E(q') < \infty, \quad E(q^2) = E(q'^2) < \infty. \qquad 1.2$$

The variables q and q' are related by the fundamental formula

$$q' = \max(q-1, 0) + r, \qquad 1.3$$

or in terms of the δ notation,

$$q' = q - 1 + \delta + r, \qquad 1.4$$

where $\delta \equiv \delta(q)$ is zero for all non-zero q, and $\delta(0) \equiv 1$. It is

important to note the following consequences of the definition of the function $\delta\,(q)$:

$$\delta^2 = \delta \quad \text{and} \quad (q)(1-\delta) = q. \qquad 1.5$$

On forming the expectations of both sides of Eq. 1.4, it will be found that

$$E(\delta) = 1 - E(r) = 1 - b/a = 1 - \epsilon. \qquad 1.6$$

This is the chance that δ is non-zero; that is, it is the probability that an incoming customer will not have to wait. On squaring both sides of Eq. 1.4 and making use of Eq. 1.5, one finds that

$$- 2\delta r + r^2 + \delta(1-\delta-2r)$$

$$q'^2 = q^2 - 2q(1-r) + (r-1)^2 + \delta(2r-1). \qquad 1.7$$

$$2\{1-E(r)\}E(q) = 2\{1-E(r)\}E(r) + E(r^2) + \{1-1+E(r)-2E(r)\}$$

Take expectations and note that r is independent of q and δ. It follows that

$$\begin{aligned} E(q) &= E(r) + \frac{E\left[\overset{r^2 - r}{r(r-1)}\right]}{2\{1-E(r)\}} \\ &= \frac{b}{a} + \frac{\operatorname{var} v + b^2}{2a(a-b)}. \end{aligned} \qquad 1.8$$

Now suppose that a departing customer leaves q customers behind him, and let his own waiting time and service time be respectively w and v. Then q is the number of arrivals in a total time w + v. Thus,

$$E(q) = \frac{E(w) + E(v)}{a}. \qquad 1.9$$

Using Eqs. 1.8 and 1.9 one finds that

$$\frac{E(w)}{a} = \frac{\epsilon^2}{2(1-\epsilon)} \left\{ 1 + \mathrm{var}\left(\frac{v}{b}\right) \right\}, \qquad 1.10$$

$$\frac{E(w)}{b} = \frac{\epsilon}{2(1-\epsilon)} \left\{ 1 + \mathrm{var}\left(\frac{v}{b}\right) \right\}. \qquad 1.11$$

This is essentially Kendall's result. It is convenient to express the result in the form of the two ratios, for in applying the results to aircraft delays, one usually normalizes time in units of the mean arrival interval or mean service time (minimum safe landing interval).

When the mean arrival interval and the mean service time are equal to each other ($\epsilon = 1$), the expected waiting time is infinite, thus showing that the intuitive solution $\epsilon = 1$ is not the best result.

From Eqs. 1.10 or 1.11 one can see that the minimum expected waiting time will be obtained if the service time distribution is of the form

$$F(v) = 0 \quad (v < b); \quad F(v) = 1 \quad (v \geq b), \qquad 1.12$$

that is, the service time is constant. On the other hand, if the service time is of the form

$$dF(v) = e^{-v/b} \frac{dv}{b}, \quad 0 < v < \infty \qquad 1.13$$

the expected waiting time is twice the minimum value ($\mathrm{var}\frac{v}{b} = 1$).

It is tempting to think that the expressions for E(q) and E(w) may be valid for more general input processes; but this seems to be a fallacy. The difficulty is that r and q are no longer statistically independent.

Kendall shows that the Markov chain associated with the queueing process is irreducible and the states are aperiodic. A further classification of the states depends on the value of the traffic intensity, ϵ. When ϵ is less than unity, all states are ergodic; when ϵ is greater than unity, all states are transient; in the critical case when ϵ is equal to unity, the states are all recurrent and null. (Feller 9)

1.5.2 Aircraft delays.

Bowen and Fearcey (6) give an elementary analysis of the effect of control procedures on the flow of air traffic, while Fearcey (7) presents a more rigorous treatment of the problem and derives the distribution of delays under different traffic conditions. The hypothesis of random arrivals is used, so that one is led to the negative exponential distribution for the arrival intervals. See Eq. 1.1.

The assumption is made that the minimum safe landing interval (service time) is constant. Thus we have Kendall's solution with a constant service time in a different notation.

Fearcey's treatment is based on a rather complicated procedure of successive integrations but once the assumption is made that the system is in a state of statistical equilibrium the analysis must lead to the same results. One feature of interest is the derivation of the probability of successive delays. By successive integrations Fearcey finds that in general the probability of n-1 aircraft being successively delayed is (stack of n-1 aircraft)

$$P(n) = A(n) T^{n-1} e^{-nT},$$
$$A(n) = -\sum_{r=1}^{n} (-1)^r \frac{(n-r)^r}{r!} A(n-r),$$
$$A(0) = 1,$$
$$T = \epsilon.$$

Bell (5) gives a somewhat more general treatment by using a moment generating procedure.

1.6 General independent input.

The center of interest of Lindley's (3) paper is the waiting times of the customers. A relation between the waiting times of successive customers is found, and this makes it possible to calculate the waiting time distribution of any customer.

This relation is also used to obtain necessary and sufficient conditions for the existence of a stationary (equilibrium) state. The paper concludes with an investigation into the stationary state in some special cases; particularly in the case when the customers arrive at regular intervals.

1.6.1 Waiting time distribution.

The queue here is again of the type with a single server but with general service time and input distributions; the intervals between successive arrivals may have any suitable distribution.

Let t_r be the time interval between the arrival of the r^{th} and $r + 1^{st}$ customer, and let s_r be the service time of the r^{th} customer.

The following assumptions are made:

1. The t_r are independent random variables with identical probability distributions and the mean, $E(t_r)$, is finite:

$$E(t_r) = \int_0^\infty t_r \, p(t_r) \, dt_r < \infty .$$

2. The s_r are independent random variables with identical probability distributions and the mean, $E(s_r)$, is finite.

Thus the two sets of random variables $\{S_r\}$ and $\{t_r\}$ $(r = 1, 2, \ldots)$ are statistically independent.

Let w_r be the waiting time of the r^{th} customer. It is evident that the following relations exist (Fig. 1.1):

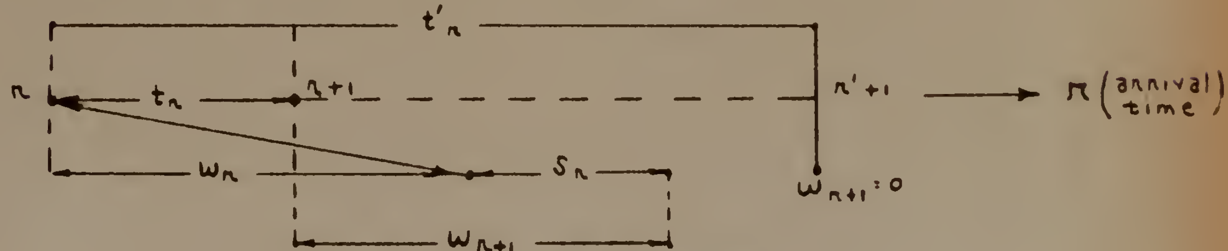


Fig. 1.1 Waiting Time Relations

$$t_r + w_{r+1} = w_r + s_r \qquad (t_r < w_r + s_r) \qquad 1.14$$

$$w_{r+1} = 0 \qquad (t_r \geq w_r + s_r;\ \text{see } t_r') \qquad 1.15$$

Let $u_r = s_r - t_r$. The u_r are independent random variables (by virtue of the proceding assumptions) and have identical probability distributions; $E(|u_r|)$ is finite and u_r is independent of w_r. Then

$$w_{r+1} = w_r + u_r \qquad (w_r + u_r > 0), \qquad 1.16$$

$$= 0 \qquad (w_r + u_r \leq 0). \qquad 1.17$$

Now if we assume that the first customer does not have to wait ($w_1 = 0$) we have the following relations:

$$w_1 = 0 \; ; \; w_2 = u_1 \; ; \; w_3 = u_2 + u_1 \; ;$$
$$w_{r+1} = u_r + u_{r-1} + \cdots + u_1 \qquad 1.18$$

and the unexpected fact develops that the waiting times depend on only the difference between the service time and interval time. If we define $F_r(x)$ as the probability that $w_r \leq x$, and $G_r(x)$ as the probability that $u_r \leq x$, we can slightly modify Lindley's procedure and find the distribution of waiting times of the $r + 1^{st}$ customer by using Eq. 1.18. The u_r are independent statistically. Thus the distribution function of $F_r + 1$ is the convolution of $u_r + u_{r-1} + \ldots\ldots + u_1$ (Doob 9) and

$$p(w_{r+1} \leq x) = \int_{-\infty}^{\infty} d\,G_1(u_1) \cdots \int_{-\infty}^{\infty} F_r(x - u_1 - \cdots - u_{r-1})\, dG_{r-1}(u_{r-1}) \qquad 1.19$$
$$= F_{r+1}(x).$$

Using the definition of $G_r(x)$, we have

$$F_{r+1}(x) = p\left(u_1 + u_2 + \cdots + u_r \leq x \; ; \; u_2 + \cdots + u_r \leq x \; ; \; \cdots\, u_r \leq x\right). \qquad 1.20$$

Eq. 1.19 establishes a recursive relation such that the individual waiting time distributions may be found. To find the final behavior of the queue, we must find the limit of $F_{r+1}(x)$ as the number of customers continually increase. Let E_r be the event:

$$u_1 + u_2 + \cdots + u_r \leq x \; ; \; u_2 + \cdots + u_r \leq x \; ; \; \cdots\, u_r \leq x \; . \qquad 1.21$$

Now the sequence of events $\{E_r\}$ decreases to a limit E as r approaches infinity, that is:

$$\lim_{n \to \infty} F_{n+1}(x) = \lim_{n \to \infty} p(E_n) = p(E) \qquad 1.22$$

by a theorem of probability measure. In the limit, then,

$$F(x) = \int_{u \leq x} F(x-u)\, dG(u) \qquad 1.23$$

$$= \int_{y \geq 0} F(y)\, dG(x-y) \qquad 1.24$$

This integral equation is difficult to solve, but it is possible to do so once G(u) is specified and certain conditions are satisfied (Paley and Weiner 11). The solution of either integral equation gives the distribution function of the waiting time, that is, the probability of waiting a time not more than x. The distribution function for any particular customer can be calculated by performing the necessary integrations; but this is also difficult to do.

Lindley uses Feller's theory of recurrent events and the strong law of large numbers to investigate the behavior of F(x).

The event of "not having to wait", called event A, is a recurrent event, since the queueing process may be considered to make a fresh start whenever a customer does not have to wait, and its previous history is quite irrelevant to subsequent developments.

To investigate the behavior of F(x) one must examine the relation

$$E(u) = E(s) - E(t) \qquad 1.25$$

as follows:

E(u) > 0: The strong law shows that the event A is transient and

that with probability 1 there will be some point in the queueing process where every subsequent customer will have to wait.

$E(u) < 0$: The strong law shows that event A is certain with a finite mean recurrence time. It should be clear that it is not periodic.

$E(u) = 0$: The event A is certain, but the mean recurrence time is infinite. Hence it is certain that there will always be another occasion on which a customer does not have to wait but the probability of any particular customer not having to wait tends to zero.

Thus a necessary and sufficient condition that the waiting time distribution function tends to a non-degenerate limit, as the number of customers increases, is that $E(u) < 0$ or $u = 0$ certainly.

Lindley solves the integral equation when the arrivals are random that is,

$$dG(y) = \lambda e^{-\lambda y} dy \; ; \; E(y) = \frac{1}{\lambda} , \tag{1.26}$$

and also for

$$dG(y) = \lambda^2 y e^{-\lambda y} dy \; ; \; E(y) = \frac{2}{\lambda} . \tag{1.27}$$

These distributions are the Type 111 curves of Pearson commonly called the gamma distributions (Mood 10), and are of the form

$$dF(y) = \frac{\lambda^{n+1}}{n!} y^n e^{-\lambda y} \; ; \; E(y) = \frac{n+1}{\lambda} \tag{1.28}$$

with the given parameters n and λ.

And as expected, the probability of not having to wait is

$$1 - \frac{E(s)}{E(t)} = 1 - \epsilon . \tag{1.29}$$

The equation to be solved for regular arrivals, where the arrival interval is the unit on the time scale, is

$$F(x) = \int_{y \leq x} F(x-y)\, dG_1(y+1) : \qquad 1.30$$

with the service time distribution as given by Eq. 1.28,

$$F(x-1) = \frac{\lambda^{n+1}}{n!} \int_0^x F(x-y)\, y^n e^{-\lambda y}\, dy . \qquad 1.31$$

The solution is difficult to develop, but many may be expressed in the form

$$F(x) = 1 - \sum_{i=1}^{n+1} c_i e^{z_i x} \quad , \qquad 1.32$$

$$c_i = \bar{c}_i \quad \text{and}$$

$$\frac{1}{\lambda^{r+1}} + \sum_{i=1}^{n+1} \frac{c_i}{(\lambda + z_i)^{r+1}} = 0 \qquad (r = 0, 1, \cdots n) ,$$

$$z_i = \bar{z}_i \quad \text{and}$$

$$\frac{\lambda^{n+1}}{(\lambda + z)^{n+1}} = e^{-z} .$$

The usual form of service time distribution is one whose frequency function is zero at s = 0, increases to a maximum and then decreases, with a long tail, to zero as s approaches infinity. In some cases it may decrease steadily from some non-zero value at s = 0. To a good approximation these functions are represented by the gamma distributions (n = 0, 1).

The results of the calculations show that regular arrivals are roughly twice as efficient as random arrivals, i.e., the probability of not having to wait for regular arrivals is roughly twice as great as the random arrivals. Lindley's calculations for a few values of ϵ are listed in Table 1 to show the nature of the results (units are arbitrary).

Probability of not having to wait			Mean waiting time	
n = 0				
ε	Regular Arrivals	Random Arrivals	Regular Arrivals	Random Arrivals
0.830	0.314	0.167	1.823	4.166
0.715	0.511	0.286	0.684	1.786
0.635	0.642	0.375	0.349	1.042
0.500	0.797	0.500	0.128	0.500
n = 1				
0.830	0.386	0.167	0.834	3.124
0.715	0.608	0.286	0.285	1.339
0.635	0.743	0.375	0.132	0.781
0.500	0.884	0.500	0.039	0.375

Table 1

1.7 Correlated input.

In view of the difficulties encountered with the random and independent inputs, it should be readily apparent that the inclusion of conditional probabilities in the input process will make the problem even more difficult. As might be expected then, this problem has not been solved analytically.

The use of the Poisson input in air traffic leads to a complete analytical solution of the stack-delay problem, but unfortunately once aircraft are in the air, the input is not even approximately random, but has the form of a regular pattern with large errors. It is virtually impossible to find the actual distribution with any accuracy, as the amount of data required for an unbiassed sample is prohibitive. The key

to the difficulty lies in the fact that aircraft in the air proceed on a predetermined schedule. This detailed information in the schedule should be regarded as a datum of the problem and used in its solution.

The first significant contribution of the electronic calculator to the queueing problem has been its use in the study of the flow of scheduled air traffic. The Research Laboratory of Electronics at the Massachusetts Institute of Technology (4) are the pioneers in this endeavor. They undertook a program to determine, as far as practicable, the quantitative relation between the degree of control of aircraft enroute to an airport, and the resulting congestion and delay.

Actually there are two possible problems involved in the control aspects of air traffic. One is the maintenance of a pre-assigned schedule, and the other is the reduction in the congestion around the terminal or landing strip. It should be noted that this is an unusual situation in that the aims of the customer and the server are the same: (a) to ensure safety in the air; (b) to allow the most efficient utilization of the landing platform; (c) to reduce the total time by avoiding long delays at the terminal. This is not always true in the general queueing problem.

In the MIT report, IBM punched-card machines were employed to make a theoretical analysis of the resulting congestion at a single landing strip when aircraft are scheduled to arrive there in some proper sequence, but fail to meet such schedules according to certain simple deviation statistics (rectangular, triangular, and parabolic). These deviations statistics are assumed to have a finite spread, that is, deviations in excess of a specific amount have zero probability. This

is within the bounds of reason from the pilot's viewpoint. Numerical results for the distribution of the resulting stack and total delays are presented. The former are compared with those which arise from the more random Poisson arrival distribution. The amounts of congestion and stack delay are found to be considerably smaller under the new conditions, particularly when the traffic is heavy. A significant range of the parameters has been covered by the numerical analysis, supported by strictly analytical methods wherever possible. Thus there is established a quantitative relation between the time-keeping errors of the aircraft enroute to an airport and the resulting terminal congestion and delay.

The density of the traffic flow to an airport is described by the usual traffic parameter ϵ, defined as the ratio of the average arrival rate at the airport (number of planes per unit time) to the maximum allowable acceptance rate. This is also the ratio of the landing interval (service time) to the arrival interval.

The authors found that it was convenient to use time in discrete units rather than on a continuous basis. The minimum safe landing interval, t_o, is chosen as the unit of time, and forms the base for both the normalization and quantization of all other times appearing in the problem.

A sample size of 1,000 planes was selected for use in the numerical analysis. In the programming procedure, each plane in the sample is represented by a card, which is identified by a number j, with j greater than or equal to 1 and less than or equal to 1000. Thus j defines the scheduled arrival order. The scheduled arrival times t_j are then as-

signed using a random number table.

Between its scheduled take-off and actual arrival times the aircraft is subject to a delay r_j, which is assigned (punched onto the j^{th} card) according to the enroute distribution used. Thus plane j actually arrives at time $p_j = t_j + r_j$, which number is also entered on the j^{th} card. It is possible now for more than one plane to arrive at the same time and for them to arrive in a sequence which differs from that scheduled. This leads to the formation of a stack of aircraft, and consequent delays while waiting in the stack. It is then assumed that planes will be handled at the terminal in order of actual arrival, with the provision that simultaneous arrivals will be considered as ordered according to their originally scheduled sequence.

After the preliminary details were completed, i.e., the t_j and r_j tables were prepared, and the initial conditions were chosen, the IBM machines were programmed in detail for the specific cases, i.e., for each deviation distribution, a spread S, and traffic parameter ϵ.

The principal results which came directly from the machines were:

(a) Frequency distributions for the stack delays.

(b) Average stack delay for each run.

(c) Cumulative frequency distributions for the stack delays.

(d) Frequency distributions for the total timekeeping errors (stack delay plus enroute delay).

(e) Average timekeeping error for each run.

(f) Cumulative frequency distributions for the total timekeeping error.

Without loss of generality, it was assumed that all variations were delays (simple change of scale).

The results form a considerable source of data pertinent to a traffic of properly scheduled aircraft. The data are given in detail in the form of curves of stack-delay probabilities, progressive-delay probabilities, and average delays, for the three types of enroute-deviation distributions considered.

The curves give additional information regarding the effects of variation of the parameters. In particular, only relatively small differences in the results are caused by rather moderate variations in the shape of the enroute-deviation distributions. The relatively small changes in the stack-delay distribution caused by making moderate changes in the shape of the enroute-deviation for given S made it possible to average together the IBM results for all three basic shapes. The average curves consequently give stack-delay probabilities for an average shape enroute-deviation distribution, which is sort of cross between the rectangular, triangular, and parabolic shapes.

The effects of changes in the traffic parameter ϵ are practically the same for all three types of distribution. For values of ϵ close to unity, the average delay changes fairly rapidly with ϵ , while corresponding changes in ϵ , when it has smaller values, cause little change in the average delay. The stack-delay distribution is also dependent in a similar way upon the traffic parameter. When ϵ is close to unity, the most probable stack delay is almost equal to its average value, but as ϵ decreases, the most probable delay rapidly approaches zero

with a small average delay. The remaining variable parameter, the spread S, also has a decreasing effect with decreasing values of ϵ.

1.8 Tabulation of cases covered.

The principal cases covered may be tabulated briefly as follows:

(a) Poisson input with general service time distribution (Kendall 2). Derives expressions for the expected waiting time and the expected queue size.

(b) General Independent Input i.e., general service time and input distributions including regular arrivals (Lindley 3). Derives the waiting time distribution of any customer and the distribution function of the waiting time in the stationary case.

(c) Correlated input i.e., scheduled air traffic (Adler and Fricker 4). Use electronic calculator (IBM) to find the frequency distributions for the stack delays, cumulative frequency distributions for the stack delays, and the average stack delay for each run. The information is presented in the form of useable graphs.

1.9 Implications

An interesting interpretation of the MIT results may be visualized in the following manner. Consider the problem of bombing when a large number of planes may be directed to strike several targets. The usual practice, to avoid saturating the handling capacity of the airfield, is to stagger the formations in such a manner that all of the aircraft will not return at the same time.

For example, consider a 200 plane strike made up of 20 groups of 10 planes per group. Now we have no idea of the enroute deviation distribution, but let us use our imagination and say that the average enroute-deviation distribution described in Section 1.7 is applicable.

Assume the expected flight time to target and back to be 10 hours for each group. Let the minimum safe landing interval be approximately 2 minutes per plane so that it takes on the average 20 minutes to land a group (t_o = 20 minutes).

It is desired that the 20 groups arrive home in such a manner that the planned maximum stack is to be 2 groups or less (20 planes). Larger stacks are not feasible due to lack of control facilities, safe altitude separation, etc.

For a 10 hour flight, errors of $\pm$ 20 minutes would not be deemed too large; in the notation of the MIT report, this will correspond to a spread of 40 minutes. If we normalize the spread in units of t_o, S = 2.

If we apply the results of the MIT paper with $\epsilon = 1$, that is, with the departure interval also 20 minutes and with S = 2, we find the probability of stacks greater than or equal to 2 groups is approximately <u>0.95</u>. In other words, it is almost certain that more than 2 groups will overlap at the field. If we reduce ϵ to 0.8, that is, make the departure interval 25 minutes, the probability that stacks greater than or equal to 2 groups will form is reduced to <u>0.06</u>. Thus it may be seen that queueing theory, as developed by electronic computers used in conjunction with good experimental data, has striking possibilities.

This example is of course trivial in that it is such a restricted

problem. But even so, the results are well worth further study. The Poisson solution when ϵ is 1 has been stressed enough; for example, the stack approaches infinity, and the waiting time approaches infinity. Thus there would be no solution in that event. When ϵ is 0.8, the Poisson solution gives for the probability of 2 or more groups in the stack a value of 0.25 approximately. In comparing the Poisson solution with the IBM solution we see the two results differ by a factor of 4, a considerable discrepancy. The key to the difficulty is the use of schedule information as a datum of the problem.

CHAPTER II

CARRIER AIRCRAFT LANDING DELAYS

2.1 Formulation of the problem.

The prime desideratum in landing aircraft aboard a carrier is simply to land the aircraft as quickly as possible while observing the necessary rules of safety. The time required to land (and launch) the aircraft aboard the aircraft carrier is of basic importance due to the extreme vulnerability of the carrier during the landing operations. The intuitive solution of the problem of minimizing the landing time is to minimize the landing intervals by feeding the aircraft into the landing pattern as quickly as possible subject to the restraint of minimum safe landing interval. While this is sometimes a satisfactory solution, yet sometimes the delays involved become excessive. Thus in attempting to minimize the landing time, one must investigate the behavior of the aircraft during the landing process. This fact makes necessary a quantitative analysis of the landing process, with a view towards a more efficient utilization of the landing platform. Thus it seems natural to apply the theory of queues in the analysis. Let us consider the main aspects of the situation under the three headings introduced by Kendall to characterize a queueing process.

As a preliminary step, consider the pattern of events around an aircraft carrier. All of the aircraft waiting to land are placed in an orbit above the carrier at a safe altitude until the signal to begin landing is given. This may be visualized as a ready-made stack (queue).

When the signal to land is given, the aircraft descend to the landing pattern to commence their approach. There is a constraint in that only a maximum number of aircraft are allowed in the landing pattern. Every time an aircraft lands it is replaced in the landing pattern by one of the waiting aircraft. Thus one can visualize the single server queueing process.

First, there is the input process. In our context this is essentially the process by which the aircraft depart the orbiting stack (queue) and feed into either the landing pattern directly or into another stack (queue) of aircraft waiting to enter the landing process. The landing pattern may also be visualized as a queue with its associated problem of delays due to wave offs, etc. This is a problem for further study and is not considered here. Thus we may visualize two queues with a single server. It should be noted that the orbiting stack (high) is called a queue in a strictly limited sense. This artifice makes the analysis more understandable. The low stack (queue) is the feature of interest in that it is caused by some perturbation in the landing process. We shall assume that all the aircraft leave the high stack at a constant rate (continuous flow), i.e., at a succession of equally spaced times, the difference between one time and the next being called the departure interval. Thus the departure distribution is of the form

$$G(r) = 0 \quad (r < v) \; ; \; G(v) = 1 \quad (r \geq v) ,$$

where r is the departure rate (constant). Second, there is the queue discipline. We shall require that the aircraft land in the order in

which they arrive at the low stack if one exists. Third, there is the service mechanism, which is given by the frequency distribution of the landing times (serving time). In a limited sense we shall consider the landing pattern as a part of the service mechanism. Landing time is here taken as the time from the cut signal given by the landing signal officer to the time the aircraft is safely forward of the barriers. We will now make the drastic simplifying assumption that the landing time is constant, i.e., the landing distribution is

$$F(s) = 0 \quad (\ell < s); \quad F(s) = 1 \quad (\ell \geq s),$$

where l is the landing rate (constant). This assumption is made for two reasons: (a) observational data are lacking, and (b) it is desired to make the analysis tractable. It seems rather natural to assume that the landing time distribution is one whose frequency function is cut off at the minimum safe landing interval, increases to a maximum near some average value of landing time, and then decreases to another arbitrary cut off point as the landing time increases. Some consideration will be given later to certain statistical implications pertaining to the gathering of observational data. A second drastic assumption to be made in connection with the landing time is that under normal conditions (when the normal landing rate is in effect) the aircraft are landed aboard without any delay for a certain length of time, and then the normal landing rate is changed abruptly, and continues at a new reduced constant rate for a period of time T_0. The reason for this reduction in rate may be poor handling technique aboard the carrier, inexperienced

pilots, etc. During this period of time the low stack begins to form. It is now easily seen why we consider two queues in series with a single server.

An important parameter of the problem is the number N of planes initially in the high stack. Under normal conditions (when the low stack is empty) the time for landing N aircraft is

$$T_r = N/r.$$

Thus the problem is to investigate the delays under the two conditions,

$$T_o \geq T_r \quad \text{and} \quad T_o \leq T_r.$$

Before proceeding with the analysis it should be noted that the fundamental difference between the general queueing problem and the problem at hand is that we are not interested in the limiting case, that is, in an infinitely long queueing process. The stack (queue) associated with the carrier problem has a finite bound N as a datum of the problem, and this must be included in the analysis. However, as the departure interval approaches the landing time, we might expect from the work of Lindley that the situation is somewhat critical for short queues as well as for long ones. This is in fact the case.

The notation to be used is as follows:

$\lambda(t)$: landing rate aboard aircraft carrier (planes per unit of time).

$r(t)$: departure rate from high stack (planes per unit of time).

$\epsilon = \frac{r}{\lambda}$: normal traffic density parameter also equal to the ratio of the landing time to the departure interval.

α : landing rate reduction parameter.

T_o : total time α is diminishing the landing rate.

T_s : total time during which non-zero low stack exists.

T_r : total time for carrier to complete landings under normal conditions.

N : number of planes initially in high stack

S(t): number of planes in low stack (low stack size).

T_{ac} : total additional time required for carrier to complete landings due to formation of low stack.

τ : aircraft delay.

D : total aggregate delay (say in plane-minutes) equal to product of average delay ($\bar{\tau}$) and the total number of planes delayed; also equal to the product of T_s and S(t).

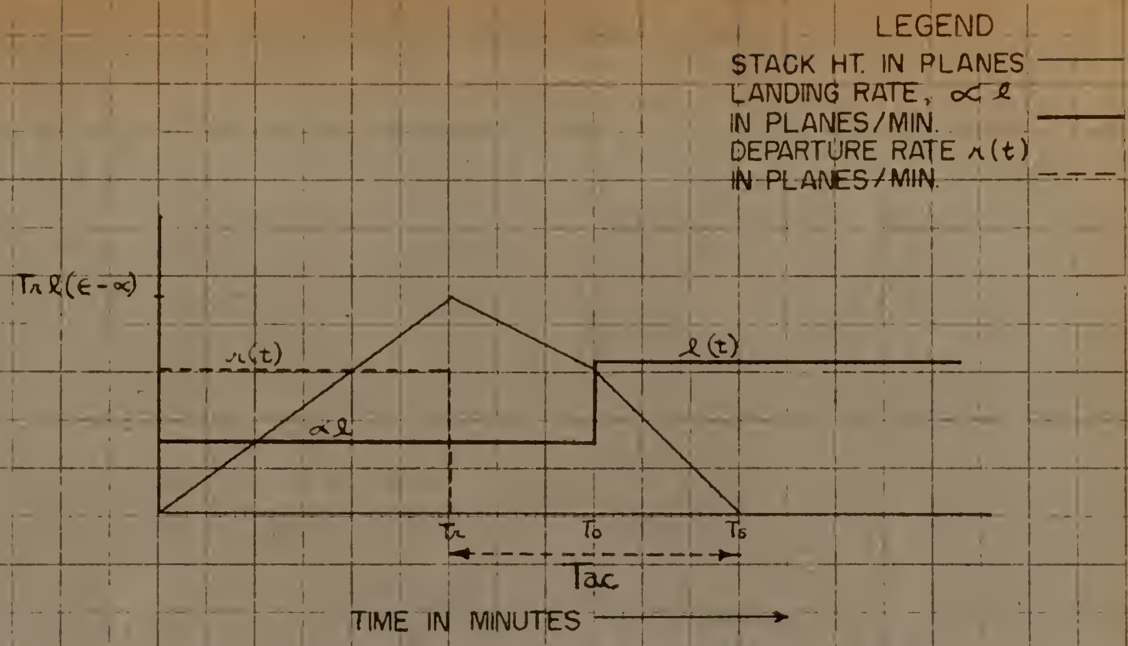
Note that the traffic density ϵ is not a function of time.

The numerical quantities of primary interest will be: (1) the additional time required for the carrier to complete landings due to the reduction of the landing rate; (2) the average stack delay over the period during which a non-zero low stack exists; and (3) the maximum stack delay suffered by any plane.

The calculations fall into two major groups, defined by $T_o \geq T_r$ and $T_o \leq T_r$. The first case to be considered will be $T_o \geq T_r$. Figs. 2.1, 2.2, 2.5, and 2.6 show the corresponding departure rate, landing rate, and stack height for prescribed values of α and ϵ in the ranges

$$0 < \alpha < \epsilon < 1.$$

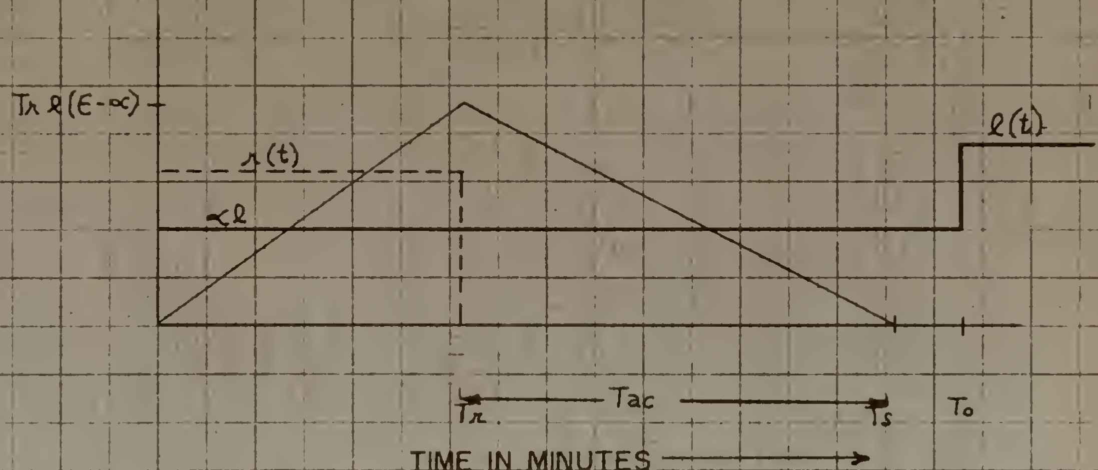
We can assume without any loss of generality that T_o commences when t = 0. Prior to this time, the landing process is normal, say ($\epsilon = \frac{n}{\ell}$; $\epsilon \leq 1$) and there is no low stack. It is not essential to



LANDING RATE, DEPARTURE RATE, AND STACK HEIGHT VS TIME PLOTS

$\epsilon < 1$

FIGURE 2.1



LANDING RATE, DEPARTURE RATE, AND STACK HEIGHT VS TIME PLOTS

$\epsilon < 1$

FIGURE 2.2

the analysis whether the landing process is in progress at $t = 0$, or just commencing.

Now at the beginning of time T_0 the landing rate is reduced to $\alpha \ell$ and, provided $\alpha < \epsilon$, a low stack begins to form at a rate $(r - \alpha \ell)$.

$$r - \alpha \ell = \ell(\epsilon - \alpha).$$

It is essential to keep in mind that no stack forms at all unless $\alpha < \epsilon$, and this condition is therefore implicit throughout the derivations. At time T_r, the departure rate from the high stack becomes zero, and, in consequence the stack no longer increases but begins to decrease at a rate $\alpha \ell$. After a further period of time $(T_0 - T_r)$, the aircraft carrier resumes its normal landing rate. As the high stack has cleared, the low stack decreases with the normal landing rate ℓ.

2.2 Stack size $S(t)$ and stack time T_s.

At time T_r the low stack height is

$$S(T_r) = T_r \ell (\epsilon - \alpha) ; \qquad 2.1$$

$S(T_r)$ is equal to $S(T)_{max.}$. See Fig. 2.1. Now the stack begins to decrease at a rate $\alpha \ell$, and at time T_0,

$$\begin{aligned} S(T_0) &= T_r \ell (\epsilon - \alpha) - (T_0 - T_r) \alpha \ell \\ &= T_r \ell (\epsilon - \alpha \, T_0/T_r). \end{aligned} \qquad 2.2$$

Now $S(T_0)$ is non-negative if and only if

$$\frac{\epsilon}{\alpha} \geq \frac{T_0}{T_r} \, ;$$

if this is not the case and

$$\frac{\epsilon}{\alpha} < \frac{T_0}{T_r} \, ,$$

the stack clears entirely prior to the end of time T_0. See Fig. 2.2.

Thus we have two cases to investigate:

Case A: $\frac{\epsilon}{\alpha} \geq \frac{T_o}{T_r} \geq 1$. Case B: $\frac{T_o}{T_r} \geq \frac{\epsilon}{\alpha} > 1$

Case A: $\frac{\epsilon}{\alpha} \geq \frac{T_o}{T_r} \geq 1$.

The time taken to clear the stack left at time T_o is

$$\frac{S(T_o)}{\ell} = \frac{T_r \ell}{\ell} \left(\epsilon - \alpha \frac{T_o}{T_r} \right)$$

$$= T_r \left(\epsilon - \alpha \frac{T_o}{T_r} \right). \qquad 2.3$$

Now the time that a non-zero low stack exists is

$$T_s = T_o + T_r \left(\epsilon - \alpha \frac{T_o}{T_r} \right). \qquad 2.4$$

Case B: $\frac{T_o}{T_r} \geq \frac{\epsilon}{\alpha} > 1$

The stack height at time T_r is

$$S(T_r) = T_r \ell (\epsilon - \alpha). \qquad 2.5$$

In this case the stack begins to decrease at a rate $\alpha \ell$; thus the time for the stack to clear is

$$\frac{S(T_r)}{\alpha \ell} = \frac{T_r \ell (\epsilon - \alpha)}{\alpha \ell}$$

$$= T_r \left(\frac{\epsilon}{\alpha} - 1 \right). \qquad 2.6$$

The time that a non-zero stack exists is

$$T_s = T_n + T_n \left(\frac{\epsilon}{\alpha} - 1 \right)$$

$$= T_n \left(\frac{\epsilon}{\alpha} \right). \qquad 2.7$$

Average low stack $\bar{S}$.

Case A: $\frac{\epsilon}{\alpha} \geq \frac{T_o}{T_n} \geq 1$

The average stack height by definition is

$$\overline{S(T)} = \frac{1}{T_s} \int_0^{T_s} S(t)\, dt$$

$$= \frac{1}{T_s} \left\{ \int_0^{T_n} t\, \ell (\epsilon - \alpha)\, dt + \int_{T_n}^{T_o} \left[T_n \ell (\epsilon - \alpha) - t \alpha \ell \right] dt + \int_{T_o}^{T_s} \left[T_n \ell \left(\epsilon - \alpha \frac{T_o}{T_n} \right) - t \ell \right] dt \right\},$$

and after integration and simplification

$$\bar{S} = \frac{\ell T_o \left[(1-\alpha)\left(2\epsilon - \alpha \frac{T_o}{T_n}\right) - \epsilon \frac{T_n}{T_o} (1 - \epsilon) \right]}{2 \left[\epsilon + \frac{T_o}{T_n} (1 - \alpha) \right]}. \qquad 2.8$$

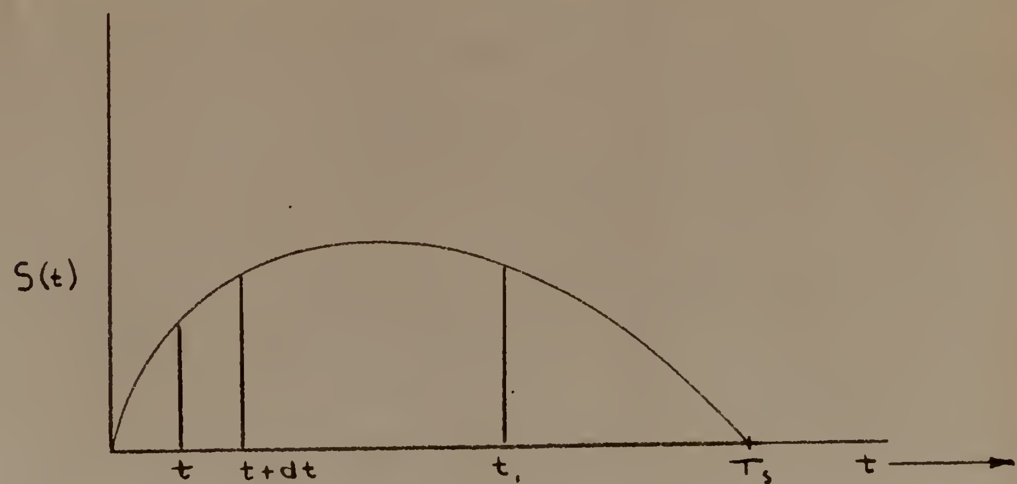
Case B: $\frac{T_o}{T_n} \geq \frac{\epsilon}{\alpha} > 1$

The average stack $\bar{S}$ is easily seen from Fig. 2.2 to be

$$\bar{S} = \frac{1}{2} T_n \ell (\epsilon - \alpha). \qquad 2.9$$

2.3 Average delay $\bar{\tau}$.

The derivation of the average delay by the usual process for computing averages is rather unwieldy. It is much more convenient to compute $\bar{\tau}$ from the ratio of the average stack height $\overline{S(T)}$ to the average landing rate $\overline{\lambda(t)}$ or the average departure rate $\overline{r(t)}$, all averages being taken over the period T_s during which the stack is never zero. That $\bar{\tau}$ can very generally be computed correctly in this manner may be demonstrated in the following way.



Stack Height vs. Time Plot for General Case

Fig. 2.3

Consider a low stack commencing at $t = 0$ and ending at $t = T_s$ as shown in Fig. 2.3. Assume the variation of $S(t)$ as arbitrary but subject to the following restraints:

$$S(0) = S(T_s) = 0, \tag{2.10}$$

$$S(t) > 0 \text{ for } 0 < t < T_s. \tag{2.11}$$

In terms of the notation previously defined, it follows from Eqs. 2.10 and 2.11 that

$$S(t) = \int_0^t [r(t) - \ell(t)]\, dt \qquad 2.12$$
$$0 \leq t \leq T_s .$$

This of course puts certain general restrictions upon r(t) and ℓ(t), since they must lead to a stack subject to Eqs. 2.10 and 2.11. Granting these general restrictions, Eq. 2.12 is valid regardless of the detailed form of either r(t) or ℓ(t). Now since $S(T_s) = 0$,

$$\int_0^{T_s} \ell(t)\, dt = \int_0^{T_s} r(t)\, dt . \qquad 2.13$$

That is, for such a stack variation, the total number of planes departing the high stack equals the total number landed.

Consider the planes r(t)dt which arrive in the time interval between t and t + dt. They will have to wait say, until $t = t_1$ to land, because of the time required to clear the stack S(t). It follows therefore that t_1 is a function of t established by

$$S(t) = \int_t^{t_1} \ell(t)\, dt , \qquad 2.14$$

which simply states that all the planes in the stack at time t must have landed at a rate ℓ(t) by the time t_1. From Eqs. 2.12 and 2.14,

$$\int_0^t [r(t) - l(t)]\,dt = \int_t^{t_1} l(t)\,dt\,,$$

$$\int_0^t r(t)\,dt = \int_0^{t_1} l(t)\,dt\,, \tag{2.15}$$

which defines t_1 as a single valued function of t. Thus as might be expected, the delay of a plane is a function of the time it arrives.

Now by definition, the average delay $\overline{\tau}$ is the total aggregate delay D divided by the number of planes delayed, where

$$D = \int_0^{T_s} (t_1 - t)\,r(t)\,dt\,.$$

That is, the planes arriving in the time interval dt are delayed a time $(t_1 - t)$ and the delays must be summed over the period T_s during which the stack is never zero. And of course the number of planes delayed is simply the total number arriving during the period T_s. Thus

$$\overline{\tau} = \frac{\int_0^{T_s} (t_1 - t)\,r(t)\,dt}{\int_0^{T_s} r(t)\,dt}\,, \tag{2.16}$$

or in view of Eq. 2.13,

$$\overline{\tau} = \frac{\int_0^{T_s} (t_1 - t)\,r(t)\,dt}{\int_0^{T_s} l(t)\,dt}\,. \tag{2.17}$$

Now the numerator can be integrated by parts as follows:

$$\int_0^{T_s} t_1 \, n(t)\,dt = \left[t_1 \int_0^t n(t)\,dt \right]_{t=0}^{t=T_s} - \int_0^{T_s} \left\{ \int_0^t n(t)\,dt \right\} dt_1 \,. \qquad 2.18$$

By Eqs. 2.10 and 2.15,

$$t_1 = 0 \quad \text{when} \quad t = 0, \qquad 2.19$$

$$t_1 = T_s \quad \text{when} \quad t = T_s. \qquad 2.20$$

This simply states that the aircraft which arrive when the stack is zero are not delayed at all. Using Eq. 2.15 to eliminate

$$\int_0^t n(t)\,dt$$

from the last term of Eq. 2.18, we have

$$\int_0^{T_s} t_1 \, n(t)\,dt = T_s \int_0^{T_s} n(t)\,dt - \int_0^{T_s} \left\{ \int_0^{t_1} \ell(t)\,dt \right\} dt_1 \,, \qquad 2.21$$

and by a similar process

$$\int_0^{T_s} t \, n(t)\,dt = T_s \int_0^{T_s} n(t)\,dt - \int_0^{T_s} \left\{ \int_0^t n(t)\,dt \right\} dt \,. \qquad 2.22$$

Thus

$$\int_0^{T_s} (t_1 - t)\, n(t)\,dt = \int_0^{T_s} \left\{ \int_0^t n(t)\,dt \right\} dt - \int_0^{T_s} \left\{ \int_0^{t_1} \ell(t)\,dt \right\} dt_1 \,. \qquad 2.23$$

Now dt_1 is merely functioning as a variable of integration in the last term of Eq. 2.23, so that

$$\int_0^{T_s} (t_1 - t)\, n(t)\,dt = \int_0^{T_s} \left\{ \int_0^t \left[n(t) - \ell(t) \right] dt \right\} dt \,. \qquad 2.24$$

Using the rather obvious definitions

$$\overline{s(t)} = \frac{1}{T_s} \int_0^{T_s} s(t)\, dt , \tag{2.25}$$

$$\overline{\ell(t)} = \frac{1}{T_s} \int_0^{T_s} \ell(t)\, dt , \tag{2.26}$$

$$\overline{r(t)} = \frac{1}{T_s} \int_0^{T_s} r(t)\, dt , \tag{2.27}$$

and Eqs. 2.12 and 2.13, we have the desired alternative form of Eqs. 2.16 and 2.17:

$$\overline{\tau} = \frac{\int_0^{T_s} \left\{ \int_0^{t} [r(t) - \ell(t)]\, dt \right\} dt}{\int_0^{T_s} \ell(t)\, dt} ,$$

and

$$\overline{\tau} = \frac{\overline{s(t)}}{\overline{\ell(t)}} = \frac{\overline{s(t)}}{\overline{r(t)}} . \tag{2.28}$$

Case A: $\quad \frac{\epsilon}{\alpha} \geq \frac{T_o}{T_r} \geq 1$

Using Eqs. 2.4, 2.26, and observing Fig. 2.1, we find that the average landing rate is

$$\overline{\ell(t)} = \frac{1}{T_s} \int_0^{T_o} \alpha \ell\, dt + \int_{T_o}^{T_s} \ell\, dt = \frac{\epsilon \ell}{\frac{T_o}{T_r}(1-\alpha) + \epsilon} . \tag{2.29}$$

Thus from Eqs. 2.28, 2.29, and 2.8, the average delay is

$$\bar{\tau} = \frac{T_o}{2\epsilon}\left[(1-\alpha)\left(2\epsilon - \alpha\frac{T_o}{T_n}\right) - \epsilon\frac{T_n}{T_o}(1-\epsilon)\right]. \qquad 2.30$$

Case B: $\dfrac{T_o}{T_n} \geq \dfrac{\epsilon}{\alpha} > 1$

From Fig. 2.2 and Eqs. 2.26, 2.28, and 2.9, the average delay is

$$\bar{\tau} = \frac{T_n}{2}\left(\frac{\epsilon}{\alpha} - 1\right) \qquad 2.31$$

2.4 Maximum delay τ_{MAX}.

In order to find the maximum delay, the delay of planes arriving at all possible times within T_s must first be considered.

Fig. 2.4 Stack Height vs. Time

As illustrated in Fig. 2.4, a group of planes arriving in the time interval $(t, \ t + dt)$, with $t \leq T_r$, finds a stack of $\ell\ (\epsilon - \alpha)t$ planes waiting to land. Now the landing rate is $\alpha\ \ell$ at any time $t < T_0$, so that the delay for the group of planes arriving during time dt is

$$\tau = \frac{t\ \ell\ (\epsilon - \alpha)}{\alpha\ \ell} = t\ \frac{(\epsilon - \alpha)}{\alpha} \qquad 2.32$$

$$0 \leq t \leq T_r .$$

Eq. 2.32 is only valid if the delay $\tau \leq T_0 - t$, for if $\tau > T_0 - t$, all of the planes will not have landed by T_0 and we have to consider the change in landing rate at T_0. Thus another limiting condition on t in Eq. 2.32, in addition to the condition

$$0 \leq t \leq T_r ,$$

is that

$$\left(\frac{\epsilon - \alpha}{\alpha}\right) t \leq T_0 - t ,$$

or

$$0 \leq t \leq \frac{\alpha}{\epsilon}\ T_0 . \qquad 2.33$$

The real limiting condition on t in Eq. 2.32 therefore depends upon whether

$$T_r < \frac{\alpha}{\epsilon}\ T_0 \qquad 2.34$$

or

$$T_r \geq \frac{\alpha}{\epsilon}\ T_0 . \qquad 2.35$$

For the condition

$$T_r \geq t > \frac{\alpha}{\epsilon} T_o \quad (\text{CASE A}), \tag{2.36}$$

the planes arriving in the interval dt will not have landed by T_o. So the number of planes still due to land before them at T_o is

$$t\,\ell(\epsilon-\alpha) - (T_o - t)\,\alpha\,\ell\,. \tag{2.37}$$

Because these planes land at a rate ℓ,

$$\tau = (T_o - t) + \frac{t\,\ell(\epsilon-\alpha) - (T_o - t)\,\alpha\,\ell}{\ell}$$

$$= T_o(1-\alpha) - t(1-\epsilon)\,. \tag{2.38}$$

Now it should be clear that the maximum delay will occur for some group of planes arriving at or just before T_r, where the stack takes on its maximum value and the landing rate remains small. The difficulties of finding the maximum near the end point of a linear equation of the form of Eq. 2.38 need not be elaborated on. The method used here is empirical and the value of t obtained to maximize Eq. 2.38 is

$$t = \frac{\alpha}{\epsilon} T_o\,.$$

Thus

$$\tau_{MAX} = \left(1 - \frac{\alpha}{\epsilon}\right) T_o$$

$$\text{FOR } \frac{\epsilon}{\alpha} \geq \frac{T_o}{T_r} \geq 1\,; \quad \text{CASE A}\,. \tag{2.39}$$

For the condition

$$t \leq T_n \leq \frac{\alpha}{\epsilon} T_o \quad (\text{CASE B}), \qquad 2.40$$

the delay for any plane in the group is given simply by Eq. 2.32, and for τ to be a maximum set

$$t = T_r.$$

Thus

$$\tau_{MAX} = T_n\left(\frac{\epsilon}{\alpha} - 1\right) \qquad 2.41$$

for $\frac{T_o}{T_n} \geq \frac{\epsilon}{\alpha} > 1$; Case B.

2.5 Additional delay of aircraft carrier T_{ac}.

It should be clear that the extra time needed by the aircraft carrier to land aircraft when a low stack exists is simply

$$T_{ac} = T_s - T_r, \qquad 2.42$$

where

$$T_r = N/r.$$

Thus T_{ac} is the additional time required for the carrier to steam into the wind at high speeds and should be made a minimum for the most efficient utilization of the landing platform.

Case A: $\frac{\epsilon}{\alpha} \geq \frac{T_o}{T_n} \geq 1$

From Eqs. 2.4 and 2.42,

$$T_{ac} = T_o(1-\alpha) - T_n(1-\epsilon). \qquad 2.43$$

Case B: $\dfrac{T_o}{T_n} \geq \dfrac{\epsilon}{\alpha} > 1$

From Eqs. 2.7 and 2.42,

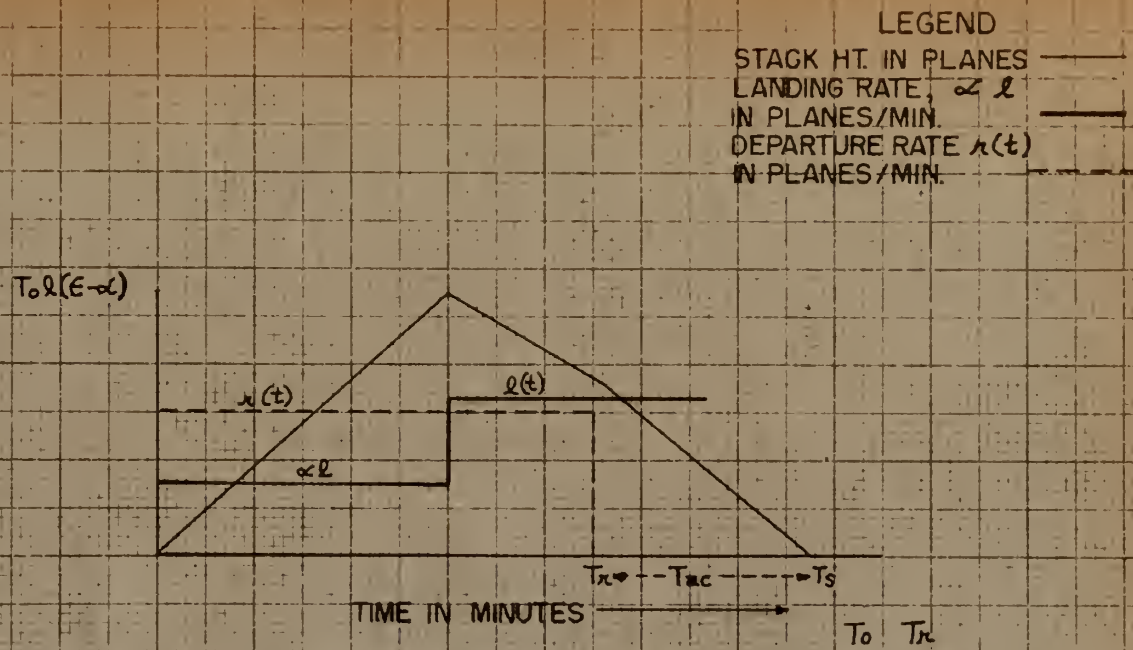
$$T_{ac} = T_n \left(\frac{\epsilon}{\alpha} - 1 \right). \qquad 2.44$$

2.6 Summary of results.

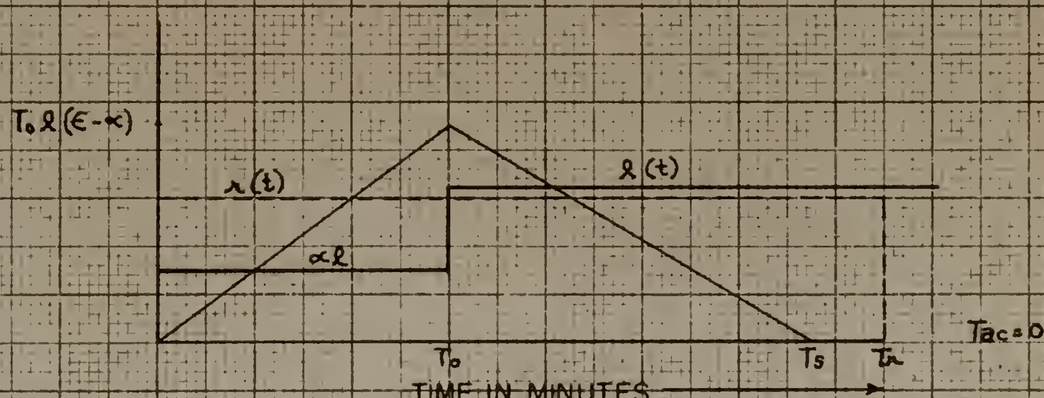
In the same manner, for $T_r \leq T_o$ the calculations split up into two portions, depending upon the time at which the stack finally clears. See Figs. 2.5 and 2.6. The complete results are summarized in Table 2, which gives the length of the stack period, the maximum stack size, the average delay, the maximum delay, and the aircraft carrier delay.

From Table 2 the aircraft carrier delay T_{ac} is seen to be in general a function of the variables T_o, T_r, α, and ϵ. It is convenient to use a normalizing factor in preparing graphs for visual representation. T_r was chosen as the normalizing factor for convenience because in practice it is usually fixed. A family of curves, for a fixed value of α, showing T_{ac}/T_r vs the variable $y = T_o/T_r$ (to specify T_o) for several values of ϵ as a parameter, were prepared. See Figs. 2.7, 2.8, and 2.9. Other curves could be prepared in a similar manner.

The general implications of the numerical results may best be appreciated in terms of examples. Consider a carrier conducting landing operations with the traffic density ϵ approximately 1. Now let us assume that during the landing operations the landing rate has been suddenly reduced from the normal rate in such a manner that when $\alpha = 0.7$, $T_o = 0.4\ T_r$; $\alpha = 0.5$, $T_o = 0.2\ T_r$; and $\alpha = 0.3$, $T_o = 0.2\ T_r$. In



LANDING RATE, DEPARTURE RATE, AND
STACK HEIGHT VS TIME PLOTS
$\epsilon < 1$
FIGURE 2.5



LANDING RATE, DEPARTURE RATE, AND
STACK HEIGHT VS TIME PLOTS
$\epsilon < 1$
FIGURE 2.6

TABLE 2
TABULATION OF FORMULAS

IN ALL CASES $\epsilon > \alpha$; NEVER $\epsilon \leq \alpha$; $T_\pi = n/\pi$

	$T_o \geq T_\pi$	$T_o \leq T_\pi$
	CASE A: $\epsilon/\alpha \geq T_o/T_\pi \geq 1$ CASE B: $T_o/T_\pi \geq \epsilon/\alpha > 1$	CASE A: $\frac{1-\epsilon}{1-\alpha} \leq T_o/T_\pi \leq 1$ CASE B: $0 < T_o/T_\pi \leq \frac{1-\epsilon}{1-\alpha} < 1$
STACK TIME T_s	CASE A: $T_o + T_\pi\ (\epsilon - \alpha\ T_o/T_\pi)$ CASE B: $T_\pi\ (\epsilon/\alpha)$	CASE A: $T_o + T_\pi\ (\epsilon - \alpha\ T_o/T_\pi)$ CASE B: $T_o\ (\frac{1-\alpha}{1-\epsilon})$
MAXIMUM STACK S_{max}	$T_\pi\ \ell(\epsilon - \alpha)$	$T_o\ \ell(\epsilon - \alpha)$
AVERAGE DELAY $\overline{\tau}$	CASE A: $T_o/2\epsilon\left[(1-\alpha)(2\epsilon - \alpha\ T_o/T_\pi) - \epsilon\ T_\pi/T_o(1-\epsilon)\right]$ CASE B: $T_\pi/2\ (\epsilon/\alpha - 1)$	CASE A: $T_o/2\epsilon\left[(1-\alpha)(2\epsilon - \alpha\ T_o/T_\pi) - \epsilon\ T_\pi/T_o(1-\epsilon)\right]$ CASE B: $T_o/2\ (1 - \alpha/\epsilon)$
MAXIMUM DELAY τ_{max}	CASE A $T_o\ (1 - \alpha/\epsilon)$ CASE B: $T_\pi\ (\epsilon/\alpha - 1)$	$T_o\ (1 - \alpha/\epsilon)$
AIRCRAFT CARRIER DELAY T_{ac}	CASE A: $T_o\ (1-\alpha) - T_\pi\ (1-\epsilon)$ CASE B: $T_\pi\ (\epsilon/\alpha - 1)$	CASE A $T_o\ (1-\alpha) - T_\pi\ (1-\epsilon)$ CASE B: 0

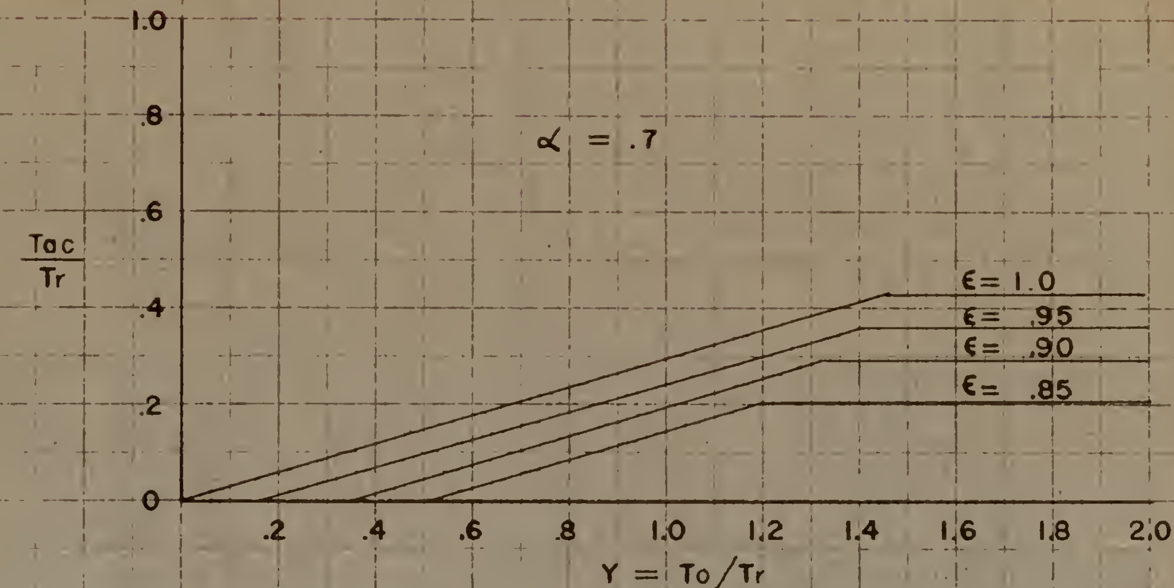
other words during landing operations the landing rate has been reduced in several instances so that on the average, say 0.8 of the time, landings were being made in an inefficient manner (poor pilot technique, inexperienced plane handlers, etc.). Now from Figs. 2.7, 2.8, and 2.9 we find that the additional delay to the carrier T_{ac} to be approximately $0.4T_r$. That is, the carrier must spend almost half the time normally taken to land aircraft steaming at high speed using extra fuel in a most inefficient manner. Taken over a period of time this could amount to a sizeable wastage, to say nothing of the danger that might be present in enemy waters.

Similarly, other values of the parameters may be used to compute new delays, and in addition the behaviour of the stack may be investigated, i.e., maximum stack, average delays, etc. These may be important in determining fuel reserves of the aircraft.

The hazards in working with the traffic density ϵ approximately 1 have been stressed throughout the paper. However, carrier landing operations are a unique application of queueing theory and it should be clear that the most efficient utilization of the landing platform will only be accomplished with ϵ approximately 1. And this can be accomplished only when the pilots and other personnel are exercising the most precise skill, judgment, etc. during the landing operations. This is of course merely a statement of what common sense would dictate.

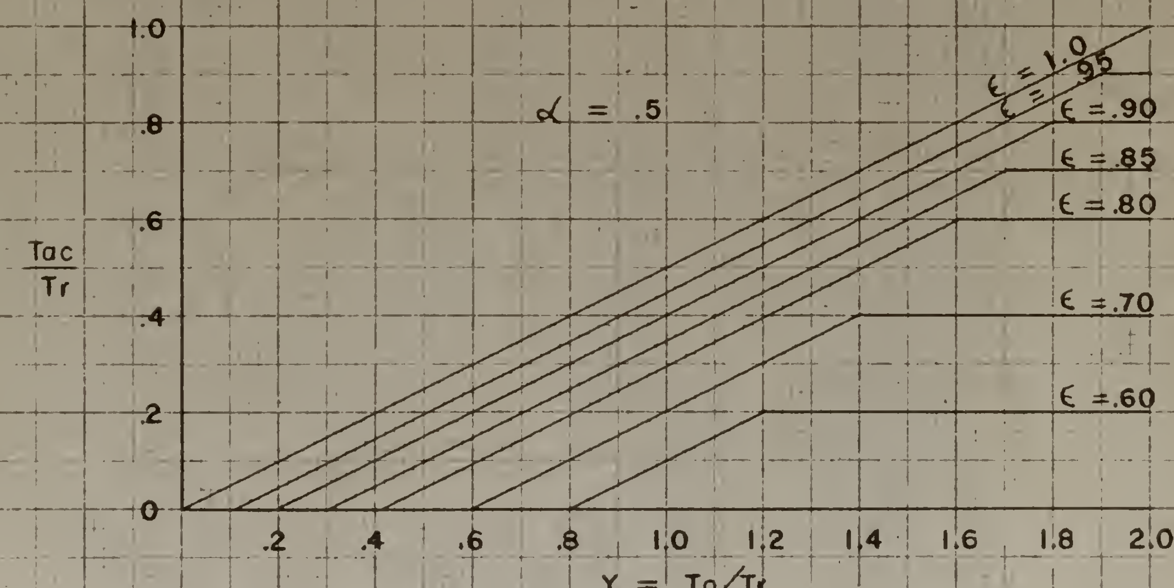
2.7 Conclusions.

For a simplified situation occuring in the landing of carrier aircraft, an investigation has been made into the variation of the average



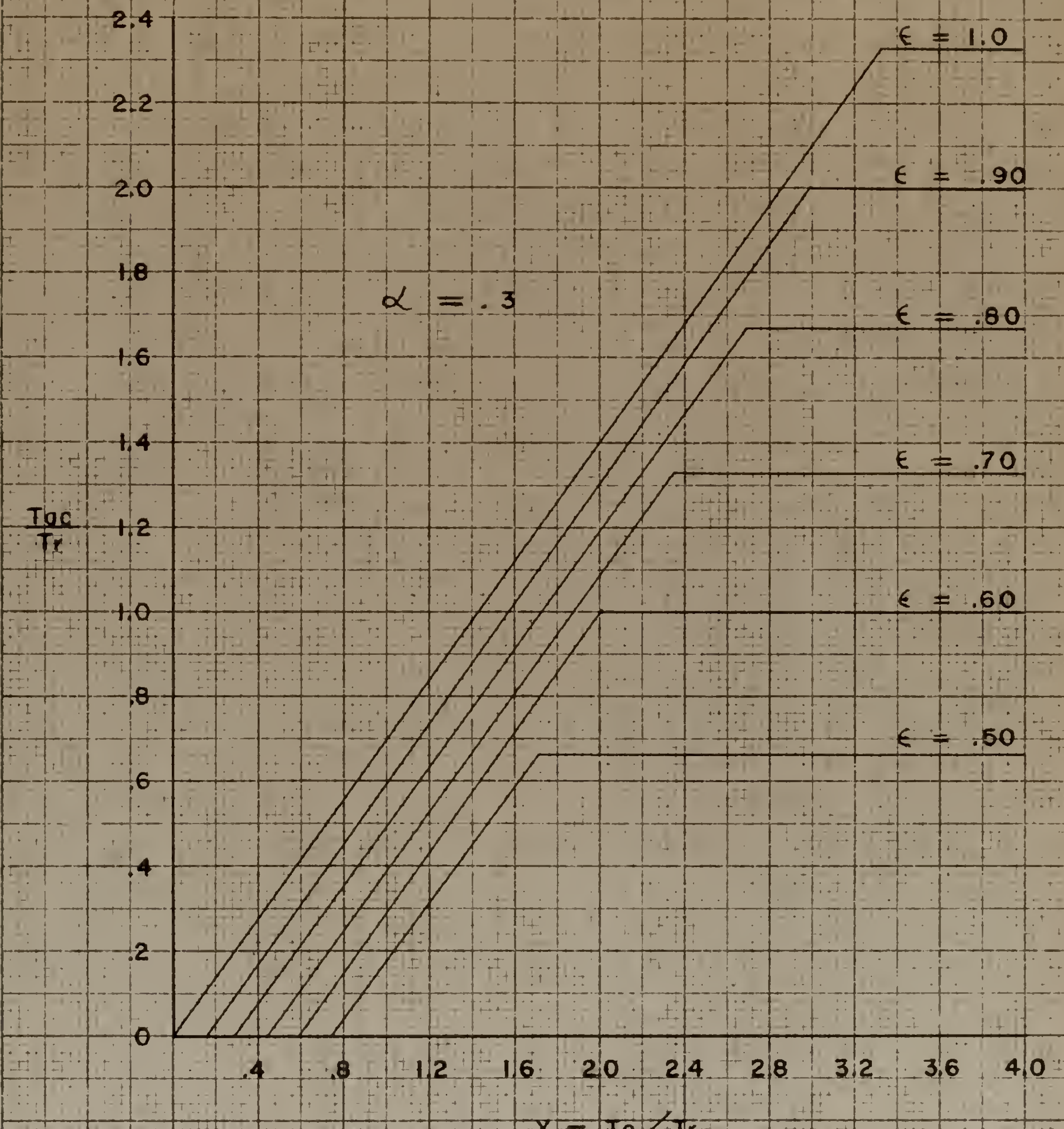
NORMALIZED CARRIER DELAY VS Y

FIGURE 2.7



NORMALIZED CARRIER DELAY VS Y

FIGURE 2.8



NORMALIZED CARRIER DELAY VS Y

FIGURE 2.9

aircraft waiting time, the time wasted by the carrier during landing operations, and the distribution of the number of aircraft in the stack for different initial conditions. The simplifications and assumptions necessary to arrive at a simple solution make the problem somewhat artificial. However, it is not completely unreasonable, because in actual practice the landing process may actually behave in some instances in a manner similar to the process assumed.

The practical verification and application of the results described in this paper depend upon the specification of the landing time distributions, the traffic density ϵ , and the minimum safe landing interval.

In comparing the present results with actual carrier operations, many observations of this process must be made during actual landings so that the validity of the assumptions made may be verified. In practice, then, a great deal of information is required to check the numerical model which has been used as a basis for the result presented here. If these assumptions are not valid as based on observational data, the model discussed in the next section may be applicable.

2.8 Statistical Implications.

The difficulties encountered in any analytical solution of a queueing process with regular arrivals and a respectable distribution of service times is apparent from Lindley's analysis (Sec 1.6). In particular, if the process is not in statistical equilibrium, the individual waiting time distributions are difficult to obtain (Eq. 1.19).

An alternative process is a quasi-emperical investigation using an electronic calculator as in the MIT report, or if that method is not available, the use of random numbers may be substituted. This is a well known standard alternative to a purely theoretical analysis, and its basic principle is as follows. If we read off successive groups of, say, four or five numbers from tables of random numbers, then these can be regarded as numbers, with four or five decimal places, drawn at random from a rectangular frequency distribution with limits 0 and 1. Now all continuous distribution functions are themselves rectangularly distributed between 0 and 1, so that the abscissa of any given distribution function corresponding to a value of the function taken from a table of random numbers can be regarded as a quantity drawn at random, with the appropriate frequency, from the frequency distribution itself. Thus for a given distribution of landing time, for example a gamma distribution with given parameters, n and λ say, we can use random numbers to run off a series of independently chosen landing times. This series can then be taken as the actual landing times of successive aircraft for a hypothetical aircraft carrier. Data for a large number of hypothetical landing patterns can be obtained in this way, and the empirical distributions,

means, standard deviations, etc., for various alternative landing systems can be worked out.

With this method of procedure it is clear that we have to decide on the basic frequency distribution of landing times beforehand, though we could consider a limited range of types of distribution (rectangular, parabolic, etc.).

In order to obtain a basic frequency distribution, observations must be made of actual landings with the landing times recorded. The usual curve fitting techniques can then be employed. Once the proper distribution has been decided upon (this may not be easy) the method of maximum likelihood may be used to find the unknown parameters from the sample data (Mood 10).

For computational ease, grouping of the frequency distribution could be utilized. From a practical point of view, a truncated distribution may be more realistic since the probability of extreme values are effectly zero. After the preliminary planning, the corresponding distribution function for each type of curve decided upon must be calculated to four decimal places. Four figure numbers are then read from the table of random numbers, and the value of x (landing time) corresponding to the mid-points of the intervals into which the numbers fell are then recorded. In this way series of landing times can be obtained readily, simply by taking successive groups of four figure numbers from the tables of random numbers and writing down the corresponding value of x read from the calculated table of the appropriate distribution function.

Now for any series of landing times the waiting time of each air-

craft and the carrier's idle or wasted time at any time may be calculated by the usual set of recursive equations (see Fig. 1.1):

$$\tau_{n+1} = \tau_n + l_n - r_n \ .$$

If τ_{n+1} is negative, that is simply the carrier's idle time. With a constant arrival interval, the computations are simple but tedious.

Finding the distribution of the actual stack is a little more complicated. The easiest method of computation seems to be to consider for how long in the intervals (assumed constant) between successive arrivals there are 0, 1, 2,....,n aircraft waiting. For example, consider a constant departure interval of 1 unit. Consider five successive aircraft who have waits of 0, 0.5, 1.5, 2, and 1 units. Assume there is no stack when the first aircraft enters the landing pattern. Thus the first aircraft doesn't wait, so there are no aircraft in the stack and it is empty for 1 unit, say. The second aircraft waits 0.5 units of the second interval, thus the stack is empty for the remaining 0.5 units. The third aircraft waits all of the third interval and 0.5 units of the fourth interval; while the fourth aircraft waits all of the fourth and fifth interval; and the fifth aircraft waits all of the fifth interval. Thus it follows that in the second interval there is one aircraft in the stack for 1 unit. In the fourth interval there are two then one aircraft waiting for 0.5 units. It is not too difficult to carry out the arithmetical operations by using tables for visual representation as follows:

Waiting time	Array		Number of a/c in stack			
			0	1	2	3
0	0		1			
0.5	0.5		0.5	0.5		
1.5	1	0.5		1	0.5	
2.0	1	1		0.5	1	
1.0	1					

The waiting times are grouped according to the arrival interval. The times during which 0, 1, ..., n aircraft are waiting during each interval are shown in the right hand table, and are obtained from successive diagonals of the center array. The figure appearing at the N. E. point of any diagonal shows how long the stack is occupied by the greatest number of aircraft in the corresponding interval. The latter number is given by the number of quantities in the diagonal. The difference between this figure and the one immediately to its S. W. gives the length of time that one fewer aircraft are waiting, etc. Subtracting the figure appearing at the S. W. point of any diagonal from 1.0 gives the length of time that the stack is unoccupied.

The above discussion explains how to obtain series of aircraft waiting times, carrier idle time, and the stack size. For any particular hypothetical carrier landing process one can calculate the mean aircraft waiting time; the total carrier idle time; and the time distribution of the stack. By considering a series of such processes it is then possible to estimate what can be expected to happen in the long run, provided the observational landing data are valid.

BIBLIOGRAPHY

1. Operational Research Quarterly Vol. 3, No. 1, March 1952.

2. Kendall, D. G., "Some problems in the theory of queues." J. Royal Statistical Society, Series B, Vol. XIII, No. 2, 1951, p. 151.

3. Lindley, D. V., "The theory of queues with a single server." Proceedings of the Cambridge Philosophical Society, Vol. 48, Part 2, p. 277.

4. Adler, R. B. and Fricker, S. J., "The flow of scheduled air traffic." MIT-RLE Reports, T. R., 198, 199.

5. Bell, G. E. (1949), "Operational Research in to air traffic control", J. Royal Aero. Soc. Oct., 1949, 965.

6. Bowen, E. G. and Pearcey, T., (1948) "Delays in air traffic flow", J. Royal Aero. Soc. April, 1948.

7. Pearcey, T., (1948) "Delays in landing of air traffic", J. Royal Aero. Soc., Dec., 1948.

8. Feller, W., "An introduction to probability theory and its applications". (1950)

9. Doob, J. L., "Stochastic Processes". (1953)

10. Mood, A. M., "Introduction to the theory of statistics", (1950).

11. Paley, R.E.A.C. and Wiener, N., "Fourier transforms in the complex domain." (1934)

12. Marshall, B. O., Jr. "Queueing theory". Informal seminar in Operations Research (1952-53), Seminar paper No. 15.

25267
Thesis
R5755
Rippa
An application of queueing
theory to carrier aircraft land-
ing delays.